Three-dimensional Modeling of Transient Electromagnetic Responses of Water-bearing Structures in Front of a Tunnel Face

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ABSTRACT

We present a finite-difference time-domain (FDTD) approach for the simulation of three-dimensional (3-D) transient electromagnetic diffusion phenomena for the detection of water-bearing structures in front of a tunnel face. The unconditionally-stable du Fort-Frankel difference discrete method is used and an additional fictitious displacement current is introduced into the diffusion equations to form explicit difference equations. We establish a new excitation loop source which considers Maxwell’s equations in source media to overcome the limitations of the precondition that the near-surface resistivity of the model is uniform in the well-known 3-D FDTD algorithm demonstrated by Wang and Hohmann in 1993. The algorithm has the ability to simulate any type of transmitting current waveforms and arbitrarily complicated earth structures. A trapezoidal wave is used to simulate a step-off source. The fictitious permittivity is allowed to vary during the computation to ensure the stability and optimize an efficient time step. Homogeneous full-space models with different resistivities are simulated and compared with the analytical solutions to demonstrate the algorithm. Transient electromagnetic (TEM) responses of a tunnel with and without a water-filled vertical fault in front of the tunnel face are simulated and compared. 3-D models with water-filled fault and karst caves in front of a tunnel face are simulated with different parameters considered.

Introduction

The transient electromagnetic (TEM) method has been widely used in near-surface geophysical exploration such as unexploded ordnance detection (Pasion et al., 2007; Doll et al., 2010; Asten and Duncan, 2012), mining exploration and monitoring (Xue et al., 2013), metallic ore exploration (Yang and Oldenburg, 2012; Xue et al., 2012), mapping contaminant migration (Pellerin et al., 2010), and groundwater investigation (Vrbancich, 2009; Ezersky et al., 2011). The TEM responses of low resistivity targets are thoroughly researched, including the very mature one-dimensional (1-D) modeling for layered earth (Nabighian, 1988; Christiansen et al., 2011), 2.5 dimensional modeling (Abubakar et al., 2006; Streich et al., 2011; Xiong, 2011) and three-dimensional (3-D) responses (Adhidjaja and Hohmann, 1989; Wang and Hohmann, 1993; Zhidanov and Tartaras, 2002; Newman and Commer, 2005; Viezzoli et al., 2008) for simple and complex models. The 3-D modeling of TEM for complex models has been implemented in the past two decades. Meanwhile, interpretation methods have also been widely researched such as the Conductivity Depth Imaging (CDI) or Conductivity Depth Transform (CDT) (Macnae et al., 1991; Huang and Rudd, 2008), Born approximation (Christensen, 1995), 1-D, 2-D, 3-D modeling and inversion (Zhidanov and Tartaras, 2002; Newman and Commer, 2005; Haber et al., 2007; Cox et al., 2010; Yang and Oldenburg, 2012; Burschil et al., 2012), pseudo-seismic migration of electromagnetic data (Zhidanov and Portniaguine, 1997; Li et al., 2005; Xue et al., 2007; Li et al., 2010; Xue et al., 2011) and principal component analysis (Kass and Li, 2012). A novel application of TEM for the prediction of water-bearing structures in front of a tunnel face has been proposed in recent years (Xue et al., 2007; Sun et al., 2011; Sun et al., 2012). It is important to ensure safety during tunnel construction, as unforeseen water in-rush threatens the safety of construction workers and tunnel structures. However, the response characteristics of TEM methods in such cases have not been investigated. The use of TEM in tunnel-face applications is still based on ground TEM theories. The application of TEM for the detection of water-bearing structures in front of a tunnel face is complex, therefore it is important to understand and...
model the response characteristics to improve TEM detection capabilities.

Generally, four methods are commonly used in 3-D TEM modeling: the integral equations (IE) method, finite-element method (FEM), finite-difference time-domain (FDTD) and finite volume (FV) method. IE is widely used in 3-D modeling of magnetotelluric (MT) and frequency-domain electromagnetic data. For TEM modeling, SanFilippo and Hohmann (1985) gave a time-domain IE solution for the TEM response of a 3-D body in a half-space model. Newman et al. (1986) improved the former research and defined the responses of a 3-D body in a layered earth. Subsequent studies simulated numerous EM problems using the IE method (Méndez-Delgado et al., 1999; Abubakar et al., 2006) and developed various improvements of this method (Hursan and Zhidanov, 2002; Zhdanov et al., 2006; Singer, 2008; Endo et al., 2008; Avdeev and Knizhnik, 2009; Zaslavsky et al., 2011). For the FEM method, the greatest advantage is the capability and flexibility in modeling arbitrarily complicated earth structures, especially using tetrahedral unstructured grids. Two-dimensional modeling of EM data using FEM has been discussed in the literature (Coggon, 1971) and is still a popular topic (Li and Key, 2007; Li and Dai, 2011). For modeling in three dimensions, Pridmore et al. (1981) introduced an investigation of FEM modeling for EM data. Variants of FEM methods were developed such as spectral-finite-element (Martinec, 1999), iterative finite-element time-domain (FETD) method (Um et al., 2012), adaptive higher order FEM (Schwarzbach et al., 2011), vector finite-element method (VFEM) (Li et al., 2011) and parallel algorithm (Puzyrev et al., 2013). FV methods in modeling EM problems were established by the Geophysical Inversion Facility of University of British Columbia (UBC-GIF) (Haber and Ascher, 2001) and were used in many inversion problems (Haber et al., 2004; Haber et al., 2007). Finite-difference (FD) or FDTD methods in EM modeling succeeded in simulating 2-D problems (Goldman and Stoyer, 1983; Oristaglio and Hohmann, 1984), but failed in simulating 3-D problems (Adhijaja and Hohmann, 1989) in the 1980’s. After that, the research on solving 3-D problems with FDTD was discussed, but developed very slowly until Wang and Hohmann (1993) presented a 3-D finite-difference time-domain solution (Wang et al., 1995; Maaø, 2007), and soon parallel algorithms were presented (Commer and Newman, 2004). Inversion techniques were also developed based on this algorithm (Wang et al., 1994; Newman and Commer, 2005). Although the number of iteration steps is large, it can be solved easily by parallel computing as FDTD has an intrinsic parallelism. The spectral Lanczos decomposition method (SLDM) described by Druskin et al. (1994, 1999) is also a novel method in solving 3-D EM problems.

We use FDTD in our simulation of 3-D TEM diffusion phenomena in tunnels. Our work improved upon the basic theory based on the well-known FDTD solution by Wang and Hohmann (1993). The unconditionally stable du Fort-Frankel difference discrete method is used, and an additional fictitious displacement current is introduced into the diffusion equations to form explicit difference equations. However, the solution from Wang and Hohmann (1993) uses the analytical EM field of a homogeneous half-space model at a very short time after the transmitting current cut off as the initial conditions. This implies a precondition that the near-surface resistivity of the model is uniform. This applies to most ground TEM surveys with an overburden, but is not applicable to TEM detection in tunnels. For a tunnel case, the area of concern is less than 100 meters in front of the tunnel face. To obtain higher resolutions of the conductivity depth profile, a high frequency is usually used. We are not interested in the responses at very late time corresponding to the conductivity far from the tunnel face. Furthermore, the complexity of a full space with an excavating tunnel cavity leads to no analytical solution. As a result, the resistivity near the tunnel face cannot be set uniformly and the analytical response of the EM field cannot be used as the initial conditions of our FDTD method. We introduce the loop current source into Maxwell’s equations by means of current density according to Ampere circuital theorem. Primary fields of TEM are included in the entire modeling and the excitation source no longer depends on assuming the near surface as a homogeneous half-space model in the initial conditions. The algorithm applies to arbitrarily complex models with non-uniform surface resistivity distributions. A trapezoidal waveform with a very short ramp time is used to simulate a step-off source. To maintain stability and optimize an efficient time step, we define the time steps using an empirical formula, also from Wang and Hohmann (1993), and then change the fictitious permittivity during the computation to satisfy the Courant-Friedrichs-Lewy (CFL) condition.

To demonstrate our algorithm, we compute several homogeneous full-space models with 3-D FDTD and compare the results with analytical solutions. In addition, we simulate the TEM responses of a tunnel with and without a water-filled vertical fault in front of the tunnel face using our 3-D FDTD algorithm. We then simulate 3-D models with water-filled fault and karst caves in front of a tunnel face. Different parameters, such as fault thickness, fault size, and resistivity contrast between the rock mass and water, are compared.
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Methodology
Maxwell’s Equations in a Source-free Medium

In lossy media such as the earth, displacement currents can be ignored for TEM exploration. Maxwell’s equations under the quasi-static approximation in linear, isotropic, lossy, non-magnetic and source-free media can be expressed as follows (Kaufman and Keller, 1983):

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (1.a)
\]

\[
\nabla \times \mathbf{H} = \sigma \mathbf{E}, \quad (1.b)
\]

\[
\nabla \cdot \mathbf{E} = 0, \quad (1.c)
\]

\[
\nabla \cdot \mathbf{H} = 0, \quad (1.d)
\]

where \( \mathbf{E} \) and \( \mathbf{H} \) are the electric field intensity and magnetic field intensity, respectively, \( \mathbf{B} \) is magnetic induction, \( \sigma \) is conductivity of the earth, and \( t \) is time.

However, the conduction current does not exist in a lossless medium such as the air in the excavated tunnel cavity. Thus, Eq. (1.b) should be modified as:

\[
\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \quad (2)
\]

where \( \varepsilon_0 \) is the magnetic permeability of a vacuum.

Diffusion equations of \( \mathbf{E} \) and \( \mathbf{H} \) can be derived from Eq. (1):

\[
\nabla^2 \mathbf{E} - \mu \sigma \frac{\partial \mathbf{E}}{\partial t} = 0, \quad (3.a)
\]

\[
\nabla^2 \mathbf{H} - \mu \sigma \frac{\partial \mathbf{H}}{\partial t} = 0. \quad (3.b)
\]

While homogeneous wave equations are derived if considering Eq. (2) as follows:

\[
\nabla^2 \mathbf{E} - \mu \varepsilon \frac{\partial \mathbf{E}}{\partial t^2} = 0, \quad (4.a)
\]

\[
\nabla^2 \mathbf{H} - \mu \varepsilon \frac{\partial \mathbf{H}}{\partial t^2} = 0. \quad (4.b)
\]

Equations (3) and (4) are approximated from homogeneous damped wave equations of \( \mathbf{E} \) and \( \mathbf{H} \) as follows:

\[
\nabla^2 \mathbf{E} - \mu \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu \sigma \frac{\partial \mathbf{E}}{\partial t} = 0, \quad (5.a)
\]

\[
\nabla^2 \mathbf{H} - \mu \varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} - \mu \sigma \frac{\partial \mathbf{H}}{\partial t} = 0. \quad (5.b)
\]

The differences lie in ignoring \( \sigma \). However, if omitting \( \varepsilon \) in Maxwell’s equations, a time derivative of the electric field will not exist. This will cause difficulty in forming an explicit FDTD discretization.

For the problem of transient electromagnetic responses of water-bearing structures in front of a tunnel face, the basic electromagnetic propagation equations are different in the excavated tunnel cavity and the rock mass. The coupling of the electromagnetic wave in the interface of the two media should also be considered.

To form the explicit difference schemes required by FDTD, we introduce the fictitious displacement current into Maxwell’s equations. Former research has proved that the solution of diffusion equations can be replaced by the solution of damped wave equations under certain conditions (Oristaglio and Hohmann, 1984).

Equation (1.b) and Eq. (2) are rewritten as follows:

\[
\nabla \times \mathbf{H} = \gamma \frac{\partial \mathbf{E}}{\partial t} + \sigma \mathbf{E}, \quad (6)
\]

where \( \gamma \) is the fictitious permittivity.

Both the conduction current and the displacement current are considered. The real permittivity is replaced by the fictitious one. The conductivity should be 0 in the area of the tunnel cavity. However, it is set to a number small enough for stability, then we can use a unified equation to solve the problem.

The difference forms of Maxwell’s equations in Cartesian coordinates and the Yee grids are the same as Wang and Hohmann (1993) and we are not going to repeat them here. Uniform grids have second-order accuracy, while non-uniform grids only have first-order accuracy because the electric field would no longer be located at the center of two adjacent magnetic field points. However, we choose non-uniform discretization of the earth and specify grids and coordinates to form a model large enough with a minimum number of cells (Fig. 1). Yu et al. (2006) recommended the ratio between two adjacent cells should be less than 1.2, and the accuracy of the result can be quite satisfying despite the loss of the second-order accuracy. The ratio of the maximum length to the minimum length of grids is limited to less than 20 to control the error resulting from the effects of the deviation from a uniform model. However, all the limitations are specified based on previous studies (Yu et al., 2006).

The low-frequency approximation involving Eq. (1.d) explicitly in Wang and Hohmann’s (1993) research calculates \( B_z \) using the value of \( B_y \) and \( B_z \) step by step upward from the bottom to the surface of the model. However, for detection in tunnels, the rock mass behind the transmitter and receiver loops should also be considered in the numerical simulation. In the detection of water-bearing structures in front of a tunnel face by TEM, the transmitter and receiver loops are put on the tunnel face, which lies in the middle of the discretization model as shown in Fig. 2. The excavated tunnel cavity is from the front to the middle part of the tunnel, which is surrounded by rock mass. The water-bearing structures may be located in front of the tunnel face between the middle to the back in the model.
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\text{2006). Hence, Eq. (6) becomes:}

\[ z \text{L}(i) \begin{cases} \text{rather than the electric field.} \\ \text{from the front to the middle} \\ \text{to calculate } B_z \text{ from the back to the middle in the } Z \text{ direction.} \\ \text{the model in the three directions are uniform.} \\ \text{that in Wang and Hohmann’s (1993) research and we give the second step} \\ \text{steps. The first step is the same as that in Wang and} \\ \text{difference iterative equations are different in the two} \\ \text{to finish a single calculation iteration step of } B_z. \]

\[
B_z^{n+1/2}(i+1/2,j+1/2,k+1) \\
= B_z^{n+1/2}(i+1/2,j+1/2,k) \\
- \Delta z_k \left[ \frac{B_x^{n+1/2}(i+1,j+1/2,k+1/2) - B_x^{n+1/2}(i,j+1/2,k+1/2)}{\Delta x_i} \right] \\
+ \frac{B_y^{n+1/2}(i+1/2,j+1,k+1/2) - B_y^{n+1/2}(i+1/2,j,k+1/2)}{\Delta y_j} \\
\]

Maxwell’s Equations in a Source Medium

We also use B_x and B_y to calculate B_z. In our case, the transmitter and receiver loops are in the middle of the model in the Z direction (as shown in Fig. 1). We first step B_z from the back to the middle in the Z direction, and then step B_z from the front to the middle to finish a single calculation iteration step of B_z. The difference iterative equations are different in the two steps. The first step is the same as that in Wang and Hohmann’s (1993) research and we give the second step difference scheme as follows:

\[
\begin{align*}
B_z^{n+1/2}(i+1/2,j+1/2,k+1) \\
= B_z^{n+1/2}(i+1/2,j+1/2,k) \\
- \Delta z_k \left[ \frac{B_x^{n+1/2}(i+1,j+1/2,k+1/2) - B_x^{n+1/2}(i,j+1/2,k+1/2)}{\Delta x_i} \right] \\
+ \frac{B_y^{n+1/2}(i+1/2,j+1,k+1/2) - B_y^{n+1/2}(i+1/2,j,k+1/2)}{\Delta y_j} \\
\end{align*}
\]

where J_s is the current density of the source.

We rewrite Eq. (8) into Cartesian coordinates as follows:

\[
\begin{align*}
\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial z} = & \gamma \frac{\partial E_x}{\partial t} + \sigma E_x + J_s, \\
\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = & \gamma \frac{\partial E_y}{\partial t} + \sigma E_y + J_s, \\
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = & \gamma \frac{\partial E_z}{\partial t} + \sigma E_z.
\end{align*}
\]

The iterative difference scheme of the electric field can be derived from Eq. (9) using a central difference scheme:

There is no analytical solution for the models of detection in tunnels. We introduce the source into Maxwell’s equations. The excitation is implemented through the source current term in Maxwell’s equation. This type of excitation is broadly used in practice because it does not cause reflection from the source region (Yu et al., 2006). Hence, Eq. (6) becomes:

\[
\nabla \times \mathbf{H} = \gamma \frac{\partial \mathbf{E}}{\partial t} + \sigma \mathbf{E} + \mathbf{J}_s,
\]

where J_s is the current density of the source.

The source position on the Yee grids is shown in Fig. 3. The source is added on the electrical field nodes. The grey grids in Fig. 3 are the elements with sources and should be processed separately. From Faraday’s law and Ampere’s law, it is easy to integrate the magnetic field at the center of each element. However, our low frequency approximation has changed the calculation of B_z, using B_x and B_y rather than the electric field.

We rewrite Eq. (8) into Cartesian coordinates as follows:

\[
\begin{align*}
\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial z} = & \gamma \frac{\partial E_x}{\partial t} + \sigma E_x + J_s, \\
\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = & \gamma \frac{\partial E_y}{\partial t} + \sigma E_y + J_s, \\
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = & \gamma \frac{\partial E_z}{\partial t} + \sigma E_z.
\end{align*}
\]

The iterative difference scheme of the electric field can be derived from Eq. (9) using a central difference scheme:
To simulate a step-off current source, we use a trapezoidal waveform with a very short ramp time. A linear ramp function is commonly used in TEM acquisition instruments. However, this waveform has four non-derivable points, marked with hollow circles in Fig. 4(a). The FDTD computation requires a smooth excitation function to minimize noise and shock effect during the EM propagation in the grids (Taflove and Hagness, 2005). We introduce a switching function to reshape the current waveform. Raised and anti-raised...
Cosine functions are used to smooth the rising edge and the ramp, respectively. The two switching functions are:

\[
U(t) = \begin{cases} 
0 & t < 0 \\
0.5[1 - \cos(\pi t / t_1)] & 0 \leq t < t_1, \\
1 & t_1 \leq t 
\end{cases}
\]  

(12)

and

\[
U(t) = \begin{cases} 
1 & t < t_2 \\
0.5 \left[1 + \cos \left(\frac{\pi t}{t_3 - t_2}\right)\right] & t_2 \leq t < t_3, \\
0 & t_3 \leq t
\end{cases}
\]

(13)

where \(U(t)\) is the current at different time \(t\), \(t_1\) is the end time of the rising edge, and \(t_2\) and \(t_3\) are the start and end time of the ramp edge, respectively.

The non-differentiable points in the trapezoidal waveform are reshaped by Eqs. (12) and (13) (see the raise and ramp edge in Fig. 4(b)). This is a smooth excitation function. To demonstrate that the frequency spectrum characteristics do not change with the switching functions, we transform the time domain functions into frequency domain to compare the amplitude and the phase (Fig. 5). The Fourier transform can be found in Appendix A.

**Stability and Boundary Conditions**

The Courant–Friedrichs–Lewy (CFL) condition is the most basic condition for FDTD problems. For a 3-D problem, the CFL condition is (Taflove and Hagness, 2005):

\[
\frac{1}{\sqrt{y_1}} \Delta t \leq \frac{\delta}{\sqrt{3}},
\]  

(14)

where \(\Delta t\) is the time step and \(\delta\) is the minimum grid length.

Reshaping Eq. (14) yields the restriction of time steps as follows:

\[
\Delta t \leq \delta \sqrt{\frac{y_1}{3}}.
\]  

(15)

We deduce from Eq. (15) that the time steps can be appropriately increased by adjusting the value of the
fictitious permittivity. This will decrease the number of iterations and save calculation time. In fact, the values of $\Delta t$ and $\gamma$ are not independent. The purpose of introducing fictitious displacement currents is to form the explicit finite difference equations. The value of the fictitious permittivity should be small enough to maintain the diffusion characteristics. We specify the time steps according to an empirical formula from Wang and Hohmann (1993) and then calculate the fictitious permittivity to satisfy the stability condition. In fact, we can also get the correct solution if using the true permittivity of the earth. The waving characteristic will disappear at the very early time, leaving only diffusion characteristics. However, this will cause the time steps to be extremely small and the iteration number to be extremely large. For example, the relative permittivity of a type of limestone is 7; if the minimum grid length is 0.5 m, the maximum time step is $2.55e-9$ seconds under Courant's condition. The calculation iterations number for 1 millisecond in pure secondary field will be 392,157 without considering the refinement of the time meshing to keep the diffusion phenomena at early times. Suitable values of the fictitious permittivity are very important to reduce the time cost.

We apply the Dirichlet boundary condition to the difference equations. Tangential components of the electric fields and vertical components of the magnetic fields at the six faces of the model are set to zero. This requires that the model must be large enough, requiring non-uniform grids.

**Numerical Examples**

To test the feasibility and the effectiveness of our methodology, we carried out several numerical simulations. A group of homogeneous full-space models are used to compare the FDTD result with the analytical results to demonstrate the accuracy and validity of the method. A model with only the tunnel cavity is simulated to show the responses of the tunnel cavity, which contains an anomalous body with high resistivity in a homogeneous full space. A model with a vertical water-filled fault in front of the tunnel face is also simulated to show the responses of TEM to water-bearing structures in front of a tunnel face.

The transmitting loop configuration and the grid meshing on the tunnel face are shown in Fig. 6. All of the related models in the paper use the same configurations. The cross section of the tunnel face is 6-m by 6-m square and the transmitting loop is 3-m by 3-m square, which is located at the center of the tunnel face. The areas near the tunnel face are meshed with uniform
grids using a 0.5-m cube. We give numberings of the grids on the tunnel face from -6 to 6 in both X and Y direction for convenience in our result analysis.

Validation using Homogeneous Full-space Models and a Half-space Model

In this part, three homogeneous full-space models with resistivities of 1 Ω-m, 10 Ω-m and 100 Ω-m are simulated for verification. We use the aforementioned transmitting loop configurations with no tunnels added.

The applications of TEM in tunnels usually use the central loop system because of the space limitation previously described. We compare the \( \frac{\partial B}{\partial t} \) at the center of the loop between the analytical solution and the FDTD solution in three dimensions (as shown in Fig. 7). The analytical solution uses a step current while
the FDTD modeling uses a trapezoidal current with a 1 \mu s ramp time. The analytical solution and the FDTD solution are in good agreement at late times.

A homogeneous half-space model with a 100 V-m background is also used to test the algorithm. A comparison of late time apparent resistivity is also given in Fig. 8.

Responses of the Tunnel Cavity

We simulate a model with only a tunnel cavity. The tunnel can be considered as an anomalous body with high resistivity inside a homogeneous full space. This model is to show the TEM responses of the tunnel cavity (as the schematic diagram shows in Fig. 9). The background resistivity is 100 \Omega-m. The conductivity of the tunnel should be 0; however, this conflicts with the stability conditions. In general practice, the conductivity of the air can be replaced with a small number (Um et al., 2012). In our simulation, the tunnel resistivity is $10^5$ times the highest resistivity of the other objects, which is $10^7$ \Omega-m in this case.

We give the decay curve responses of X, Y and Z components at different positions on the tunnel face (as shown in Fig. 9). Figure 6 gives the positions of the receiver points. The X, Y and Z components are all simulated. The four plots in Fig. 9 are representative curves. Figures 9(a)–(b) are outside the transmitting loop, while Figs. 9(c)–(d) are inside the loop. Also, Figs. 9(a)–(c) are located symmetrically at the diagonal of the transmitting loop. The X and Y components at positions both inside and outside the transmitting loop

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**Figure 10.** XZ cross-sections of $E_x$ across the axis in the Y direction of the tunnel at different decay times: (a) 10 \mu s, (b) 100 \mu s, (c) 500 \mu s, and (d) 1 ms after turn off. The white rectangle corresponds to the tunnel location. The electric fields change rapidly at the interface between the rock mass (resistivity 100 \Omega-m) and the tunnel.
should be symmetric. The X and Y components in Figs. 9(a)–(c) completely overlap; the dashed lines indicate the data are negative. The plots with X and Y components not at the diagonal position of the transmitting loop, such as Figs. 9(b)–(d), are not symmetric. The Z component response is significantly larger than the X and Y components. The inside loop position (Fig. 7(d)) has its Y component all positive, while the X component has some negative values at early time. Similar phenomena occur in its symmetrical position at point (1, −1).

The electric fields XZ cross-sections are shown in Fig. 10. The contours of Ex at four decay times (10 μs, 100 μs, 500 μs and 1 ms after turn off) are drawn. The XZ cross-section is across the axis in the Y direction of the tunnel. The white rectangle in each plot locates the edge of the tunnel. The diffusion process can be identified through the four decay times; the shape changes of the contours in front of the tunnel face and their values decrease in the computation areas. The interfaces between the tunnel and the rock mass have rapid changes of resistivity and the electric fields focus in...
those areas. The influences of the tunnel cavity on the TEM survey are obviously demonstrated by Fig. 10.

Modeling of a Water-filled Fault in Front of a Tunnel Face

The purpose of our methodology is to simulate the TEM responses of water-bearing structures in front of a tunnel face. Geophysicists can analyze the response characteristics of water-bearing structures, such as water-filled faults and underground rivers, using the 3-D FDTD modeling method. The geophysical anomalies of these water-bearing structures have a lower resistivity than the background rock mass. We present a model with a background resistivity of $100 \, \Omega\cdot\text{m}$, tunnel resistivity of $10^7 \, \Omega\cdot\text{m}$, and a water-filled fault resistivity of $1 \, \Omega\cdot\text{m}$ located 10 meters ahead of the tunnel face, as shown in Fig. 11. The size of the water-filled fault is 50-m by 50-m in the X and Y directions and 5-m in the Z direction.

With the same configurations as aforementioned, we simulate and give the decay curves at different receiving points of the three components in Fig. 11. The decay curves’ characteristics of this model are very different from the model in Fig. 9 with only the tunnel cavity—especially the X and Y components. The dashed lines in the plots indicate negative values.

Figure 12. XZ cross-sections of $E_x$ across the axis in the Y direction of the tunnel at different decay times: (a) 10 $\mu$s, (b) 100 $\mu$s, (c) 500 $\mu$s, and (d) 1 ms after turn off. In each plot, the left white rectangle corresponds to a water-filled fault while the right white rectangle corresponds to the tunnel location. The electric fields change rapidly at the interface of the rock mass and the tunnel. The contours are also quite different from that without a water-filled fault, as shown in Fig. 10.
Two direction reversals of the $\partial B/\partial t$ response exist both inside and outside transmitting loop survey points on the tunnel face. The responses on the diagonal positions of the transmitting loop are the same, while the responses not on the diagonal positions are different for the X and Y components. The low resistivity fault can be identified clearly by the Z components. We also give the XZ cross-sections of $E_X$ across the axis in the Y direction of the tunnel at different decay times (Fig. 12). The two rectangles in each plot mark the tunnel cavity (right) and the water-filled fault (left). The electric fields focus on the low resistivity fault at early times, and then change to focus on the interfaces between the rock mass and the fault, also between the rock mass and the tunnel cavity. The values change rapidly.

**Figure 13.** Response curves of the X (a) and Z (b) components at point $(-1, -1)$ on the tunnel face with different fault thicknesses. The distance between the tunnel face and the water-filled fault is 10 m. The dashed line in (a) indicates the received $\partial B/\partial t$ is negative.

**Figure 14.** Response curves of the X (a) and Z (b) components at point $(-1, -1)$ on the tunnel face with different fault thicknesses. The distance between the tunnel face and the water-filled fault is 20 m. The dashed line in (a) indicates the received $\partial B/\partial t$ is negative.
To investigate the influence of the thickness of the vertical water-filled fault, we present a group of comparisons with different thicknesses and different distances between the tunnel face and the fault. A schematic diagram is given in Fig. 13. Six models with three thicknesses and two distances are simulated. We present the response curves for the X and Z components at point (−1, −1) in Fig. 13 and Fig. 14 corresponding to distances 10 m and 20 m, respectively. The responses are quite clear as low resistivity targets. Notice that the X components have two $\frac{dB}{dt}$ reverse points in these models with the water-filled fault in front of the tunnel face.

To investigate the influence of the distance between the tunnel face and the vertical fault, we present four different models with a fixed thickness, 5 m, for the water-filled fault. The distances are 10 m, 20 m, 30 m and 50 m. The horizontal and vertical responses on the tunnel face are shown in Fig. 15. Notice the $\frac{dB}{dt}$

Figure 15. Response curves of the X (a) and Z (b) components at point (−1, −1) on the tunnel face with a fixed fault thickness (5 m) and different distances between the tunnel face and the water-filled fault. The dashed line in (a) indicates the received $\frac{dB}{dt}$ is negative.

Figure 16. Response curves of the X (a) and Z (b) components at point (−1, −1) on the tunnel face with a fixed distance (70 m) and different fault size, as given in the figure. The dashed line in (a) indicates the received $\frac{dB}{dt}$ is negative.
reverse phenomenon, and it differs slightly from the curves in Fig. 13 and Fig. 14.

To investigate the influence of the water-filled fault’s size on the TEM response, we fix the distance between the tunnel face and the fault at 70 m and give three models with different fault size. The horizontal and vertical responses on the tunnel face are shown in Fig. 16. The X components have only one $\frac{dB}{dt}$ reverse phenomenon within the simulation time. The $\frac{dB}{dt}$ reverse time is much later than the models with shorter distances, such as the first reverse time in Fig. 14 or Fig. 15. Also, the reverse times are very close to each other for the models with a distance of 70 m. The Z component responses of different anomalous body sizes exhibit similar characteristics for

Figure 17. Response curves of the X (a) and Z (b) components at point (−1, −1) with different resistivity ratios (5, 10 and 100). In this comparison, the basic model is similar with that in Fig. 13 while the distance is 10 m and the fault size is 50-m by 50-m with a thickness of 5 m. The resistivity of the rock mass and the water are also different than that in Fig. 13, but use the ratio shown in the legend. The dashed line in (a) indicates the received $\frac{dB}{dt}$ is negative.

Figure 18. Response curves of the X (a) and Z (b) components at point (−1, −1) on the tunnel face with different fault dips. In this comparison, the length of the fault is fixed at 50 m. The distance between the tunnel face and the middle right edge of the water-filled fault is 50 m; the fault size is 50-m by 50-m with a thickness of 2 m. The dashed line in (a) indicates the received $\frac{dB}{dt}$ is negative.
the low resistivity target, with the response increasing as target size increases.

To investigate the influence of resistivity differences on the TEM response, we compare three groups with different resistivity ratios (5, 10, and 100). The distance is 10 m and the fault size is 50-m by 50-m with a thickness of 5 m. The responses curves are given in Fig. 17. The anomalous response becomes more obvious when the resistivity ratio between the rockmass and water-bearing structure is large. We noticed that when the resistivity ratio reduces to 5, the Z component response is very close to the response of the model with only a tunnel cavity; meanwhile, there still exists notable differences in the X component.

To investigate the influence of a dipping water-filled fault on the TEM response, we compare three different dips (26.5°, 45°, and 56.5°) with the distance fixed at 50 m and thickness fixed at 2 m (Fig. 18). In Fig. 19, the fault length varies, but the fault projection is fixed at 50 m (as shown in Fig. 18). The response is dependent on the size of the fault. When the fault length is fixed (Fig. 18), the $\partial B/\partial t$ reverse phenomenon in the X component disappears when the dip angle is less than 45°. However, when the fault projection is fixed (Fig. 19), the responses in the X component are quite different with different dips. Differences in the X and Z components in both Fig. 18 and Fig. 19 are very small.

![Figure 19. Response curves of the X (a) and Z (b) components at point (−1, −1) on the tunnel face with different fault dips. In this comparison, the length of the fault varies while its projection is fixed at 50 m, as shown in Fig. 18. The distance between the tunnel face and the middle right edge of the water-filled fault is 50 m; the fault thickness is also 2 m. The dashed line in (a) indicates the received $\partial B/\partial t$ is negative.](attachment:fig19.png)

![Figure 20. Schematic diagram of (a) water-filled and (b) semi-water-filled karst cave in front of a tunnel face.](attachment:fig20.png)
Modeling of a Water-filled Karst Cave in Front of a Tunnel Face

In karst areas, a karst cave is frequently encountered during tunnel construction. We present the responses of water-filled and semi-water-filled karst caves with different parameters. The schematic diagrams of water-filled and semi-water-filled karst caves in front of a tunnel face are shown in Fig. 20.

To investigate the influence of karst cave size on the TEM response, we first simulate the water-filled karst cave models (Fig. 21), and then simulate a comparison model with a semi-water-filled karst cave.

Figure 21. TEM response curves for different karst cave sizes in front of the tunnel face. The distance between the tunnel face and the right side of the karst cave is 30 m. We present two different karst cave sizes for comparison, a 10-m cube and 30-m cube. The dashed line in (a) indicates the received $\frac{\partial B}{\partial t}$ is negative. The $Z$ component (b) for the 10-m cube karst cave is quite weak and cannot be distinguished from the tunnel-only model decay curve on a log-log coordinate plot.

Figure 22. TEM responses curves of a water-filled and semi-water-filled karst cave in front of the tunnel face. The karst cave size is a 30-m cube. The distance between the tunnel face and the right side of the karst cave is 30 m. The dashed line in (a) indicates the received $\frac{\partial B}{\partial t}$ is negative. The $Z$ component (b) responses of a semi-water-filled karst cave is weaker than a full water-filled karst cave. However, the abnormal response of the X component (a) is evident.
The Z component responses are nearly identical to the tunnel-only cavity model when the water-filled karst cave size is 10-m by 10-m by 10-m or smaller (Fig. 21), and the semi-water-filled karst cave size is 30-m by 30-m by 30-m or smaller (Fig. 22). A more distinctive response is seen in the X component, where the reverse phenomenon is not observed in either water-bearing structure until the cave size reaches 30-m by 30-m by 30-m. Hence, we deduce that the $\frac{\partial B}{\partial t}$ reverse phenomenon is related to the size ratio of the water-bearing structures in the X and Z components.

To investigate the influence of resistivity differences on the TEM response for a karst model, we present four comparisons with resistivity ratios of 100, 10, 5 and 2.5. The simulation results are given in Fig. 23. For karst cave models, the Z component response curves are almost the same as the tunnel-only cavity model when the resistivity ratio is less than 10; however, differences in the X component can still be clearly identified, even with a small resistivity ratio of 2.5 (see inset in Fig. 23(a)).

Results and Discussion

Our modified FDTD algorithm for modeling 3-D TEM problems in tunnels considers both source and source-free media. The diffusion phenomenon of TEM is simulated. From the results of the numerical modeling (Figs. 13-17 and Figs. 18-23), we obtain some useful TEM characteristics for identifying water-bearing structures in front of a tunnel face as follows: 1) the X component responses have two $\frac{\partial B}{\partial t}$ reverse phenomena when the water-bearing structure in front of a tunnel face is sufficiently large and the distance between the tunnel face and the structures is sufficiently small; 2) the Z component responses have clear characteristics if the low resistivity targets are sufficiently large; and 3) the response of the X component is more sensitive than that of the Z component if the target’s size is not very large or the target is located far from the tunnel face.

If we take the influence of the tunnel cavity into consideration, the conductivity of air should be set to zero. However, this will bring instability into our equation even if an extremely short time step is given. We use a resistivity large enough to replace the infinite air conductivity to conquer this instability. Thus, the equations can be solved. However, the time step must be very short as the phase velocity of the electromagnetic wave in air is much faster than that in a very lossy medium, and therefore it causes a very long computing time. We suggest two possible ways to overcome this problem. First, a parallel algorithm is recommended based on the current methodology. Second, the alternating-direction implicit finite-difference time-domain (ADI-FDTD) algorithm for time domain electromagnetic modeling may be helpful to save time in computation (Taflove and Hagness, 2005; Yu et al., 2006).
Conclusions

We have developed a finite difference time domain (FDTD) approach for the simulation of 3-D TEM diffusion phenomena in tunnels. The method applies to TEM detection in tunnels with water-bearing structures in front of a tunnel face. Arbitrarily complex models can be simulated by the methodology described in this paper. All three components of the TEM response both inside and outside the transmitting loop can be simulated. This method can also be applied in airborne TEM modeling for arbitrary complex models.

Acknowledgments

The authors would like to thank the helpful suggestions and discussions from Tsili Wang during the development of the modified FDTD modeling algorithm in three dimensions. The authors want to thank the Associate Editor, Antonio Menghini, and two anonymous reviewers for their valuable comments and useful suggestions that helped to improve the presentation of this paper. This research is funded by the National Program on Key Basic Research Project (973 Program) under the grants 2013CB036002 and 2014CB046901, the National Natural Science Foundation of China (NSFC) under the grants 51139004 and 41174108.

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APPENDIX A

FOURIER TRANSFORM OF THE CURRENT WAVEFORM BEFORE AND AFTER SWITCHING PROCESS

The current waveform functions before and after the switching process can be expressed as follows, respectively:

\[
U_1(t) = \begin{cases} 
0 & t < 0 \\
\frac{t}{t_1} & 0 \leq t < t_1 \\
1 & t_1 \leq t < t_2 \\
\frac{t - t_3}{t_2 - t_3} & t_2 \leq t < t_3 \\
0 & t \geq t_3 
\end{cases}, \quad (A.1)
\]

and

\[
U_2(t) = \begin{cases} 
0 & t < 0 \\
0.5 [1 - \cos(\frac{\pi t}{t_1})] & 0 \leq t < t_1 \\
1 & t_1 \leq t < t_2 \\
0.5 \left[ 1 + \cos(\frac{\pi t}{t_3 - t_2}) \right] & t_2 \leq t < t_3 \\
0 & t_3 \leq t 
\end{cases}. \quad (A.2)
\]

We take Eq. (A.2) as an example. Its Fourier image function is as follows:

\[
F_2(\omega) = \int_{-\infty}^{+\infty} U_2(t) e^{-i\omega t} dt. \quad (A.3)
\]

Substituting piecewise function (A.2), we obtain:

\[
\int_{-\infty}^{+\infty} U_2(t) e^{-i\omega t} dt = \\
\frac{1}{2} \int_{t_1}^{t_2} [1 - \cos(\frac{\pi t}{t_1})] e^{-i\omega t} dt + \int_{t_1}^{t_3} e^{-i\omega t} dt + \int_{t_1}^{t_2} \left[ 1 + \cos(\frac{\pi t}{t_3 - t_2}) \right] e^{-i\omega t} dt. \quad (A.4)
\]

Solve Eq. (A.4) using the quadrature rule to obtain the frequency domain expression:

\[
F_2(\omega) = \\
2t_2^2 \omega^2 [\sin(\omega t_1) + i\cos(\omega t_1) - \frac{\pi^2}{2} \sin(\omega t_1) - i(1 - \cos(\omega t_1))] \\
\frac{2(\omega^2 t_1^2 - \pi^2 \omega^2)}{2i\omega} + 2e^{-i\omega t_1} - 2e^{-i\omega t_2} - e^{-i\omega t_3} + e^{-i\omega t_2}. \quad (A.5)
\]

Similarly, we get the frequency domain Eq. (A.1) as follows:

\[
F_1(\omega) = \frac{1 - e^{i\omega t_1} + i\omega t_1}{\omega^2 t_1 e^{i\omega t_1}} + \frac{e^{-i\omega t_1} - e^{-i\omega t_2} - e^{-i\omega t_3}}{\omega^2 (t_2 - t_3)}. \quad (A.6)
\]